

## Realistic Mathematic Education: a theoretical methodological approach to the teaching of mathematics in countryside schools

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**ABSTRACT.** The movement for a Rural Education still lacks investigations of methodological theoretical assumptions for the didactic field, based on the study of teaching practices that consider the object of knowledge and, at the same time, value the realistic/contextual aspect in which the student is inserted. From this perspective, we investigate the methodological theoretical implications of the theory of Realistic Mathematical Education (EMR) for the teaching of mathematics in the countryside school. Based on a qualitative methodological approach, a hypothetical learning path was elaborated based on the principles of EMR related to the teaching of analytical geometry, from the practice of soil modeling in passion fruit (*passiflora edulis*) cultivation. Our results point to the EMR as a promising methodological theoretical approach of didactic exploration to the countryside context capable of promoting formal reasoning, concepts in realistic situations, appropriation of mathematical language and potential for the development of concepts in the field of Cartesian geometry.

**Keywords:** Realistic Mathematics Education, Emerging Models, Analytical Geometry, Countryside School, Rural Education.

## Educação matemática realística: uma abordagem teórico-metodológica para o ensino de matemática nas escolas do campo

**RESUMO.** O movimento por uma Educação do Campo ainda carece de investigações de pressupostos teórico-metodológicos para o campo didático, pautadas no estudo de práticas de ensino que considerem o objeto de conhecimento e, ao mesmo tempo, valorize o aspecto realístico/contextual onde o aluno está inserido. Nessa perspectiva, investigamos as implicações teórico-metodológicas da teoria da Educação Matemática Realística (EMR) para o ensino de matemática na escola do campo. Baseados em uma abordagem metodológica qualitativa, elaborou-se uma trajetória hipotética de aprendizagem fundamentada nos princípios da EMR relacionada ao ensino de geometria analítica, a partir da prática de gabaritação de terra no cultivo do maracujá (*passiflora edulis*). Nossos resultados apontam a EMR como uma via teórico-metodológica promissora de exploração didática para o contexto do campo capaz de promover raciocínios formais, conceitos em situações realísticas, apropriação de linguagem matemática e potencial para o desenvolvimento de conceitos no ramo da geometria cartesiana.

**Palavras-chave:** Educação Matemática Realística, Modelos Emergentes, Geometria Analítica, Escola Rural. Educação do Campo.

## **Educación Matemática realista: un enfoque teórico metodológico para la enseñanza de las matemáticas en las escuelas rurales**

**RESUMEN.** El movimiento para una Educación del Campo aún carece de investigaciones de supuestos teóricos metodológicos para el campo didáctico, basados en el estudio de prácticas de enseñanza que consideran el objeto del conocimiento y, al mismo tiempo, valoran el aspecto realista / contextual en el que se inserta el estudiante. Desde esta perspectiva, investigamos las implicaciones teóricas metodológicas de la teoría de la Educación Matemática Realista (EMR) para la enseñanza de las matemáticas en la escuela del campo. Basado en un enfoque metodológico cualitativo, se elaboró un camino de aprendizaje hipotético basado en los principios de EMR relacionados con la enseñanza de la geometría analítica, a partir de la práctica del modelado del suelo en el cultivo de maracuyá (*passiflora edulis*). Nuestros resultados apuntan a la RME como un enfoque teórico metodológico prometedor de la exploración didáctica en el contexto rural capaz de promover el razonamiento formal, los conceptos en situaciones realistas, la apropiación del lenguaje matemático y el potencial para el desarrollo de conceptos en el campo de la geometría cartesiana.

**Palabras clave:** Educación Matemática Realista, Modelos Emergentes, Geometría Analítica, Escuela Rural. Educación del Campo.

## Introduction

The rural education movement lacks investigations addressing methodological/theoretical assumptions based on the study of teaching practices that consider the object of knowledge and also value realistic/contextual aspects familiar to the student. Few teaching practices for mathematics are available to assist the teacher in the classroom at rural schools in order to implement the principles of rural education.

From the standpoint of academic performance, this scenario is critical and requires interventions, as students at rural schools in Brazil have a lower average performance in mathematics than students from urban schools. Data released by the *Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira* (INEP [Anísio Teixeira National Institute of Educational Studies and Research]) show that only 6% of students in the 5<sup>th</sup> and 9<sup>th</sup> grades of rural schools perform adequately in mathematics, which is half the rate reported for urban schools (INEP, 2011). More recent results from a large-scale evaluation also show this disparity in the level of proficiency between students at urban and rural public schools, with an average difference of 28.69 points (INEP, 2018).

To produce research aimed at assisting in the teaching of mathematics at rural schools in order to exert an impact on the performance of the students, we bring to the debate the assumptions of the theory of Realistic Mathematics Education (RME), highlighting it as a promising means of teaching in the rural context. This study presents the concepts and foundations of RME, followed by methodological procedures and a discussion of the main findings. We aim to establish RME as a theoretical/methodological approach to the teaching of mathematics at rural schools to be explored by teachers in the classroom as well as by researchers in the field of rural education.

## Realistic Mathematics Education

The instructional design theory of RME had its origins in the Netherlands in the 1970s during a universal effort to improve mathematical thinking. It is based on the interpretation of Hans Freudenthal, who conceived mathematics as a human activity (Freudenthal, 1983; Gravemeijer, 1994). In some ways, RME resembles Decroly's "centers of interest" (Gravemeijer 1994, 1999; Gravemeijer & Terwel, 2000).

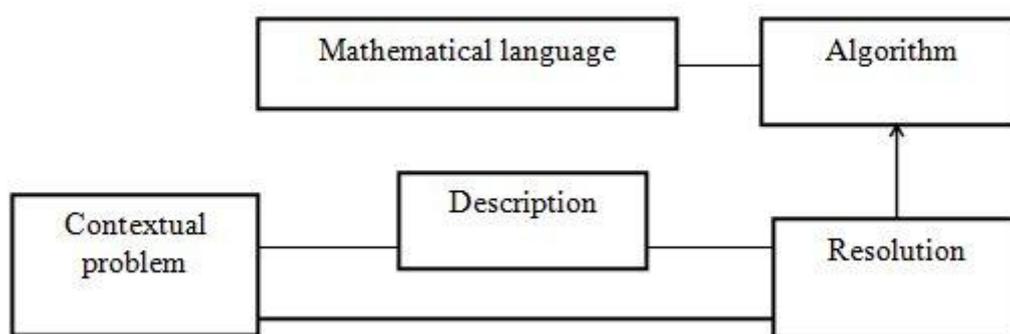
From Freudenthal's perspective, students should learn mathematics through

a process of progressive mathematization<sup>1</sup> based on real or mathematically authentic contextual problems. In this regard, RME theory is primarily a knowledge-building proposition; it does not focus on motivating students in everyday life contexts but on providing experientially real contexts to be used in the progressive mathematization process (Gravemeijer, 1999). According to Rasmussen & Blumenfeld (2007), “RME is aimed at enabling students to invent their own reasoning methods and solution strategies, leading to a stronger conceptual understanding”. (p. 198).

The aim of contextual problems is a process of reinvention on the part of students to deal with formal mathematics. It should be realistic from standpoint of

providing elements to imagine, turn into ideas and become real in the minds of students. This suggests that contextual problems need not be authentically real but must be imaginable, achievable and conceivable (Van Den Heuvel-Panhuizen, 2005; Ferreira & Buriasco, 2016). Students are able to extract information from a contextual problem and use informal strategies by trial-and-error to solve the problem. This level in RME is denominated horizontal mathematization. The translation of this information into a mathematical language using symbols and progressing to the selection of algorithms, such as an equation, is denominated vertical mathematization (Figure 1). It is a process involving the resolution of the problem situation at different levels.

Figure 1. Horizontal and vertical mathematization.



Source: Gravemeijer (2004).

Contextual problems are considered key elements in RME and must be able to form concepts and models (Treffers & Goffree, 1985). Based on De Lange (1987), Ferreira and Buriasco (2015, p.

457) classify contextual problems in first, second and third order according to the objectives and mathematical potential, as follows:

**Zero-Order Context:** This is used to make the problem look like a real-life situation, denominated by De Lange (1999) as a “false context” or “camouflage context”. Problems with this type of context should be avoided. **First-Order Context:** This presents “textually packaged” mathematical operations, in which a simple translation of the statement into a mathematical language is sufficient (DE LANGE, 1987). This type of context is relevant and necessary to solve the problem and evaluate the response. **Second-Order Context:** This is one with which the student is faced with a realistic situation and is expected to find mathematical tools to organize, structure and solve the task (De Lange, 1987). According to De Lange (1999), this type of context

involves mathematization, whereas problems are already pre-mathematized in first-order contexts. **Third Order Context:** This enables a “conceptual mathematization process”. This type of context serves to “introduce or develop a concept or mathematical model”. (De Lange, 1987, p. 76, emphasis added).

Advancing the understanding of the fundamentals of RME beyond contextual problems and their classifications, Treffers (1987) defined five principles for RME (Table 1).

Table 1. Principles of Realistic Mathematics Education.

Phenomenological exploration	Mathematical activity is not initiated from the formal level but from a situation that is experientially real for the student.
Use of models and symbols for progressive mathematization	The second principle of RME is to move from the concrete level to the more formal level using models and symbols.
Use of students' own construction	Students are free to use and find their own strategies for solving problems as well as developing the next learning process.
Interactivity	The students' learning process is not only individual but also a social process.
Interconnection	The development of an integrative view of mathematics, connecting various domains of mathematics can be considered an advantage within RME.

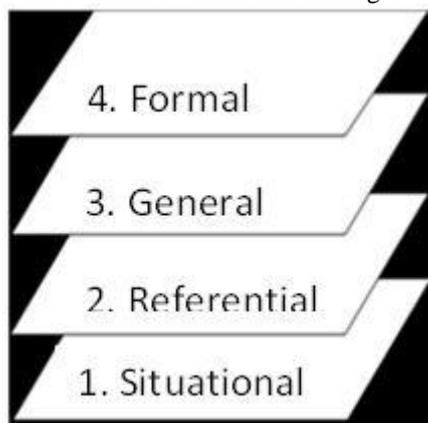
Source: Based on Treffers (1987).

A central heuristic of RME encompassing all these principles is denominated "emerging models", which can promote ways of reasoning in students for the development of formal mathematics (Gravemeijer, 1999). Zandieh and Rasmussen (2010) define models as ways of organizing an activity, whether from observable tools, such as graphs, diagrams and objects, or mental tools, referring to

the ways in which students think and reason while solving a problem (Treffers & Goffree, 1985; Treffers 1987, 1991; Gravemeijer, 1994). Models are denominated emergent in the sense that the various ways of creating and using tools, graphics, analysis and expressions emerge concomitantly with increasingly sophisticated forms of reasoning.

The emerging-models heuristic involves four levels for the development of this mathematical reasoning, starting with the situational and moving toward the formal, as shown in the figure below:

Figure 2. Levels of emerging models: from situational to formal reasoning.



Source: Gravemeijer (2004).

The intention is that a student's mathematical activity at each level changes from a contextual solution (model of) to a more general solution (model for) (Gravemeijer, Bowers & Stephan, 2003). The **situational level** is the basic level of emerging models, where students work towards mathematical goals within a contextual problem. At this level, students can use their own symbolism and related models, regardless of the conceptual mathematical rigors and the configuration of the contextual problem. The **referential level** involves building models based on the initial configuration of the task. The initially conceived models are adjusted according to the contextual problem. At the **general level**, the models built do not

depend on the configuration of the original task. Finally, the **formal level** involves reasoning with conventional symbolism that reflects a new mathematical reality in view of the contextual problem initially posed.

In summary, the models appear in specific contexts and refer to concrete, experiential and real situations for students associated with the RME principle of “phenomenological exploration”. At this level, the models must enable informal strategies to solve the contextualized problem. From then on, the model changes its role and the students can establish mathematical relationships and strategies related to the principle of “using models and symbols for progressive mathematization”. Consequently, the model becomes more objective and closer to the level of formal mathematics. Thus, RME argues that the modeling of informal mathematical activities develops a more formal mathematical reasoning model, which can improve the level of mathematical understanding and the teaching-learning process. This issue becomes critical when we consider rural education, one of the pillars of which is precisely the insertion of the context experienced by the student in the learning process.

## Methodological Path

The research takes a qualitative approach following the theoretical/methodological assumptions of the instructional design theory of RME based on the heuristic of emerging models. The instructional design was developed for teaching analytical geometry topics to students in the 3<sup>rd</sup> year of rural high school.

The research involved the exploration of the phenomenon of preparing a plot of land<sup>ii</sup> for planting passion fruit through discussions and video footage. The rural property where the investigation took place was located

around a settlement project in the municipality of São João do Araguaia (state of Paraíba, Brazil) and belonged to a local resident who extracted fruit pulp as an economic activity to acquire extra income.

The actions were performed considering the needs of the class, working in the perspective of presenting a support task to achieve the learning objectives and thus develop concepts of analytical geometry, as shown in the table below:

Table 2. Hypothetical learning trajectory for teaching analytical geometry according to assumptions of RME.

Learning objective	Concepts	Support Task
Understand elements of the Cartesian plane	Point, plane and axes	Observe the process of preparing a plot of land for planting passion fruit and model the situation mathematically.
Represent points on the Cartesian plane	Coordinated pairs	Observe the process of preparing a plot of land for planting passion fruit and model the situation mathematically.
Set distance and alignment between points	Distance between two points and alignment between three points	Second-order contextual problem

Source: research data (2019).

The support tasks were linked to the principles of RME considering our learning objectives. These tasks consisted of the presentation of the practice of preparing a plot of land for planting passion fruit, from which the second-order

contextual problem was constructed, according to De Lange (1987).

## Intervention

The intervention carried out in the classroom followed the seven steps defined below:

1- Presentation of video footage about passion fruit planting to the students, configuring a farming practice with emphasis placed on the phenomenon of driving stakes at equidistant points. The video was produced to compose the teaching material to be used in the classroom.

2- Moment of discussions for students to express orally what they understood about the phenomenon in order to provide mental constructions of the first mathematical relations.

3- Bring problematization to the students' discussions on the geometric knowledge in the preparing of the plot of land aiming to develop at a more abstract level the concepts of point, plane, axis, Cartesian plane and coordinate pairs.

4- Creation of a model in groups representing the phenomenon of preparing a plot of land for planting passion fruit as a way for students to put into practice the geometric knowledge developed in the previous steps.

5- Problematizing the geometric concepts under study based on the representative model of preparing the plot of land, thus approaching a more general level of school mathematics.

6- Demonstration in the framework of the formulas necessary to calculate the distance between two points and the alignment of three points through problematizations contextualized to the phenomenon under study.

7- Development of questions about distance between two points and the alignment of three points from the contextual problem.

## Results and Discussion

The development of the concepts of analytical geometry emerged through the contextual problem of the measurement of a plot in the practice of planting passion fruit produced by the teacher, who took on a mediating role in the process. The subsequent problem is classified as a second-order contextual problem in the terms proposed by De Lange (1987), in which the student is confronted with a realistic situation and is expected to find mathematical tools to organize, structure and solve the task.

## Contextual Problem

Before growing, passion fruit seedlings need to be planted next to wooden stakes with uniform spacing and wire stretched between the tops of the stakes following a single direction so that the passion fruit grows easily. This method

of following a uniformity forms a grid, stake to the other.  
with the equivalent distances from one

Figure 3. Forming a grid: left, stakes are driven into soil at uniform distances; right, plants ready for harvesting.



Source: research data (2019).

During the grid-forming procedure, the stakes are placed at two meters from each other to facilitate the development of plants until the harvesting of the fruit.

From the exploration of this specific characteristic of passion fruit planting, information was collected to serve as a basis for the studies, fostering the first emerging organizational models. Next, we analyze the phases of the instructional design from the levels of the emerging models (situational, referential, general and formal) as they were achieved by the students in order to focus on the contribution of the process to the transition from intuitive, informal reasoning to more sophisticated, formal modes of reasoning.

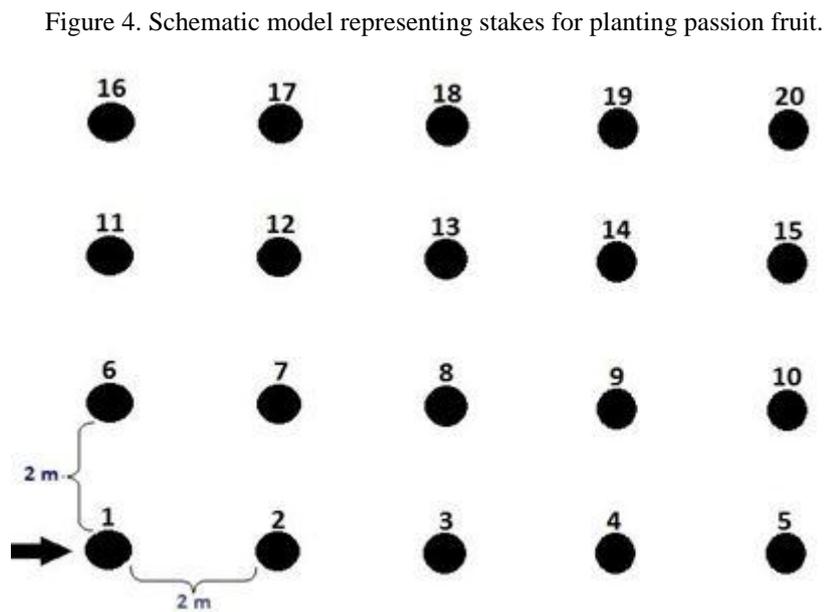
### Situational Level

Considering the emerging models, the situational level was obtained from the students' engagement in the initial observation of the educational video, translating it into a contextual problem. This was a phenomenological exploration in terms of RME principles, in which the situations are experientially real for students and designed to support a conceptual formalization process based on informal reasoning. As a rich activity in the generation of horizontal mathematization, the students were asked to watch and describe the environment of the passion fruit plantation, seeking general mathematical entities that would later be discussed. In this instructional design, conventional mathematics can be reinvented, creating an opportunity for progressive mathematization.

## Reference Level

The students were asked to observe the arrangement of the stakes in the soil considering the visualization of the video and to construct a representation of it. The model was defined by the students and each stake was represented by a “black

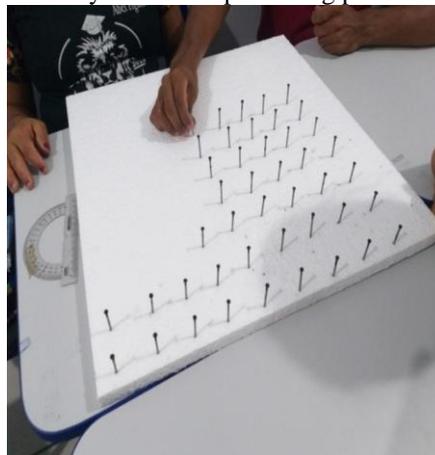
ball” numbered from 1 to 20, imagining an aerial view of the passion fruit plantation. The model was that of a small plantation with 20 stakes placed equidistant from each other at two-meter intervals, as shown below:



Source: research data (2019).

The following figure shows the model developed by the students during the classroom intervention:

Figure 5. Schematic model built by students representing positioning of stakes in grid format.



Source: research data (2019).

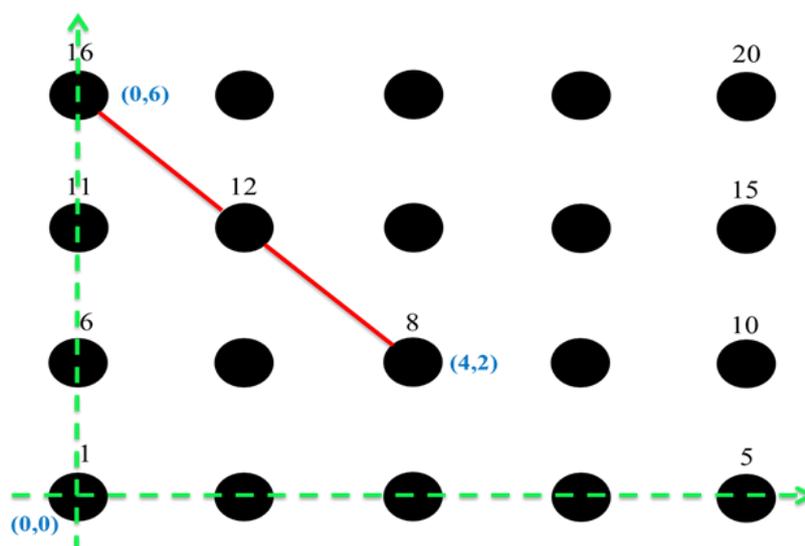
From this construction, the students achieved the referential level with the construction of a schematic model based on the initial configurations of the contextual problem: the positioning of stakes for planting passion fruit.

### General Level

Based on the previous schematic model, the students were asked to imagine these

points on a plane with Cartesian axes, expanding the idea of the situational model to a more general model. Thus, the points began to be understood in a Cartesian perspective – a model that no longer depended only on the initial contextual problem, reaching the general level of emerging models in the evolution of mathematical reasoning.

Figure 6. Schematic model considering plane with Cartesian axes.



Source: research data (2019).

In this schematic model, the land represents the plane and each stake represents a point. With the help of a system of axes associated with a plane, a pair of coordinates was matched to each point on the plane and vice versa.

### Formal Level

The approach adopted to reach the formal level, according to the emerging

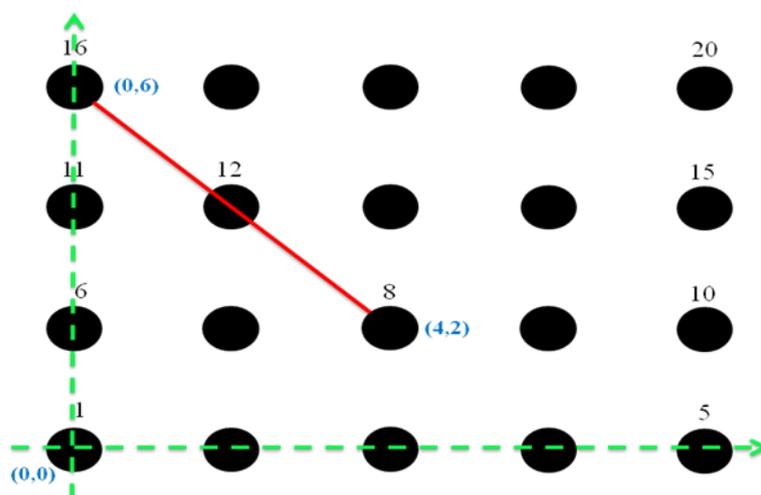
models, started from problem situations based on the initial contextual problem. At the formal level, the students reasoned about the concepts of analytical geometry using conventional notations with a reinvention guided by the teacher. For such, the previously discussed two topics of analytical geometry were addressed during the classes to provide material to support the investigative process.

The names of the subjects listed in the following problems are fictitious, since all activities in this stage were carried out in the classroom and the students had previous knowledge on the analytical geometry topics studied during the intervention.

Supposing that we intend to determine the distance between Ismael and Tiago, two employees who work on the passion fruit plantation harvesting the crop. When you know that they are positioned on stakes 16 and 8, respectively, as we can see in the figure, what would this distance be?

**Problem situation 1- Distance between two points:**

Figure 7. Schematic model of distance between two points.



Source: research data (2019).

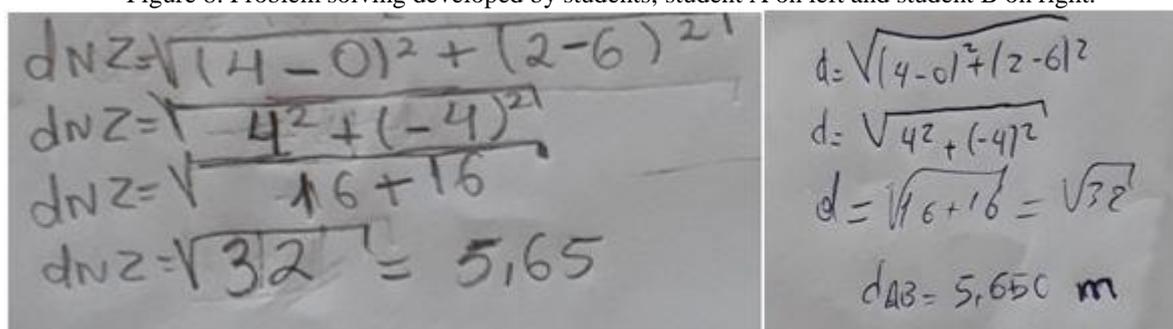
When drawing two imaginary axes corresponding to the ordinate and abscissa axes in the representation of the passion fruit plantation, the students were asked about the distance between Ismael and Tiago, as shown in the problem situation, who were at points (0,6) and (4,2), respectively.

Knowing that the distance between two points is given by the equation:

$$d_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2},$$

The students developed the problem solving as follows:

Figure 8. Problem solving developed by students; student A on left and student B on right.



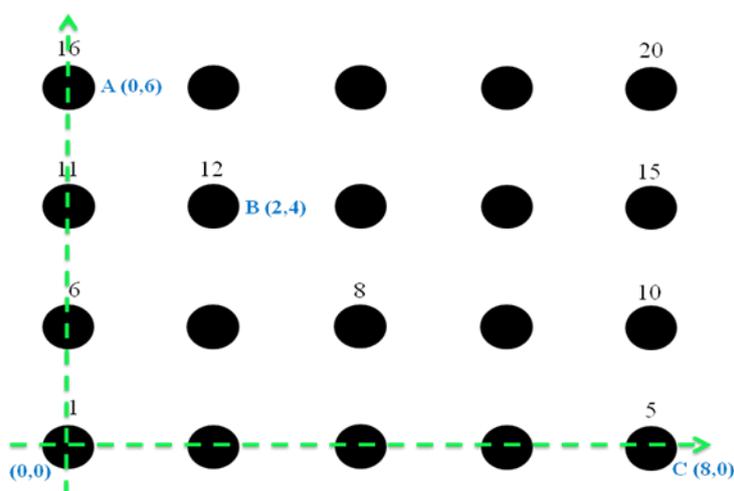
Source: research data (2019).

When carrying out the calculations, the students obtained the answer 5.65; so, the distance between Ismael and Tiago is 5 meters and 65 centimeters. The students' resolution of this problem situation shows that the sequence of actions promoted formal reasoning and the appropriation of mathematical language, as demonstrated by the correct application of the distance formula between two points by the students.

### Problem situation 2- Three-point alignment

Imagine this situation: Ismael, Tiago and Mateus are harvesting the passion fruit and each is at a certain stake. Ismael is at stake 16, Tiago is at stake 12 and Matthew is at stake 5. The employees have to work in line for a better yield, thus reducing the harvesting time and increasing productivity. We will help Ismael, Tiago and Mateus by checking if they are aligned or not on the passion fruit plantation.

Figure 9. Schematic model of three-point alignment.



Source: research data (2019).

We know that from three points A  $(x_A, y_A)$ , B  $(x_B, y_B)$  and C  $(x_C, y_C)$ , they will

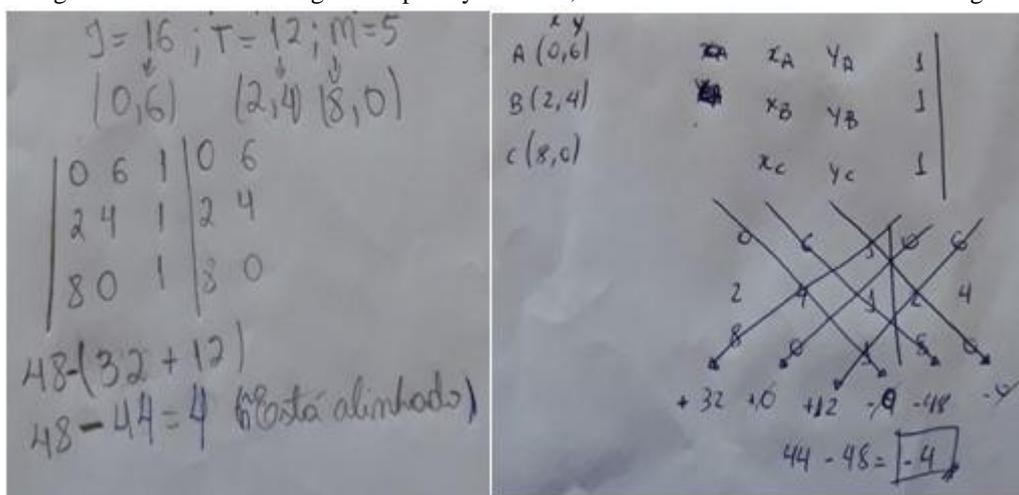
be aligned if, and only if, 
$$\begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} =$$

0. The students were able to identify the three points that represented the stakes where the employees were positioned, as shown in Illustration 10: stake 16 (0,6),

stake 12 (2,4) and stake 5 (8,0). To verify that these three points were aligned considering the positioning on the plantation, the students developed Sarrus' Rule in the matrix formed from the points:

$$\begin{vmatrix} 0 & 6 & 1 & 0 & 6 \\ 2 & 4 & 1 & 2 & 4 \\ 8 & 0 & 1 & 8 & 0 \end{vmatrix}$$

Figure 10. Problem solving developed by students; student A on left and student B on right.



Source: research data (2019).

As the answer, the students obtained the value of the determinant equal to 4, which does not satisfy the alignment condition, as its value would have to be equal to zero ( $\det = 0$ ). In this case, the students managed to abstract key concepts in the field of Cartesian geometry based on the realistic situation and also managed to advance in the understanding of resolution rules, demonstrating that the actions have the potential for the development of concepts to teach mathematics at rural schools.

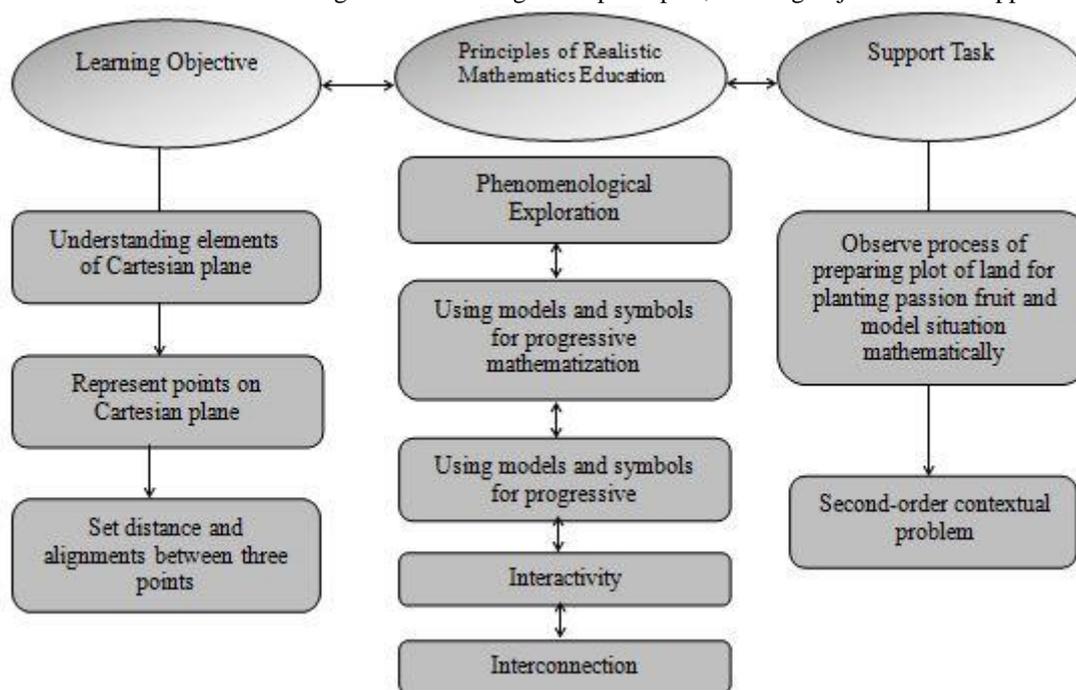
Analyzing the two problem situations described above, we can see that students were able to use the appropriate mathematical language to arrive at the result of the problem, extracting information and using informal strategies (horizontal mathematization), followed by the use of conventional mathematical symbols and algorithms (vertical mathematization).

## Links with theoretical and methodological assumptions of RME

During the intervention in the classroom, the students were instructed to build models in the terms proposed by Zandieh and Rasmussen (2010), as the

concepts of the Cartesian plane, the distance and alignment between points and the relation between axis and pairs of coordinates were developed in the classroom. The flowchart below illustrates the relations established in the instructional design.

Figure 11. Flowchart demonstrating relations among RME principles, learning objectives and support tasks.



Source: research data (2019).

In analyzing the flowchart, we see that the RME principles were achieved

during the classroom intervention, as shown in the following table:

Table 3: Principles of RME and how each was achieved during classroom intervention.

Principles of RME	How it was achieved
Phenomenological exploration	In the exhibition of the video tutorial presenting the phenomenon under study (measuring land in the practice of planting passion fruit), the students had contact with an experientially real situation.
Use of models and symbols for progressive mathematization	From the moment that the students began to develop the geometric concepts under study from the exploration of the contextual problem, especially during discussions and problematizations about the theme, enabling a natural evolution of knowledge.
Use of students' own construction	In exploring the phenomenon under study during the construction of the model, as the students could organize themselves in different ways; and in discussions and problematizations where students' knowledge began to reach more formal levels.
Interactivity	In the process of carrying out all actions planned for the classroom, with

	greater emphasis during the construction of the representative model of preparing the land, instigating students to apply the geometric knowledge in development from a propitious situation to provide the approximation of more general models of mathematics.
Interconnection	When students needed to mobilize other domains of mathematics to solve the problems of calculating the distance between two points and alignment between three points, using resources such as the formula to calculate the distance in relation to the Pythagorean theorem and the determinant calculation through 3 x 3 matrices (Sarrus' Rule) to determine the alignment of the points. Thus, in the study of analytical geometry, interconnection is as a tool present in all processes when combining algebra with geometry.

Source: research data (2019).

Linking our actions to the principles of RME, the contribution was positive for the teaching process of the topics of analytical geometry, as it has its own methodology that goes beyond traditional teaching methods, bringing the mobilization of mathematical knowledge present in sociocultural practices into the classroom.

## Conclusion

Two specific mathematical practices were promoted throughout the empirical approach: i) the construction of a point model on a plane, considering the stakes used when planting passion fruit; ii) reasoning about the Cartesian plane, including the relation of axes with pairs of coordinates. An interconnection between several mathematical domains was favored, such as the use of matrices and determinants, and primitive concepts of plane geometry, some naturally emerging from the nature of the mathematical object in question: analytical geometry as the

study of a plane and spatial geometry in an algebraic perspective.

These mathematical practices arose through contextual problems made possible by the learning environment designed by RME. This environment enabled the students to produce their own ideas in an examining and interactional process in order to evolve from levels of informal understanding to formal reasoning. In these terms, this initial contribution shows that RME can favor mathematical practices and objectives and develop conceptual understanding.

Specifically in addressing the contextual problems developed, other aspects stand out: (1) a textual model with educational purposes conceptualized in realistic situations; (2) the formulation of mathematical schemes made possible through statements; and (3) the promotion of methodological teaching/learning to explain and solve problems. Such aspects indicate RME and the heuristic of emerging models to be a significant instructional design for teaching and

learning analytical geometry at rural schools, surpassing traditional teaching approaches in terms of mathematization.

As the methodological paths used by the teacher in the classroom affect the academic performance of the students, the fundamental principles of RME in the learning environment of emerging models directly favor mathematical knowledge, promoting more sophisticated and formal reasoning. Therefore, teaching analytical geometry from the context of socio-cultural practices developed in the students' communities is relevant, as analytical geometry at most rural schools is presented in a technical, abstract way.

The teaching of mathematics in rural communities gives rise to a gamut of realistic contextual problems, which makes it possible to value the intrinsic mathematical knowledge of the daily activities of the different social groups and merits greater theoretical and methodological appropriation by educators. Thus, Realistic Mathematics Education is a promising way of teaching/learning exploration.

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<sup>i</sup> From a pedagogical perspective, mathematizing is exploring contexts in order to understand / organize / mathematically.

<sup>ii</sup> A land process template or process for defining or preparing land with equidistant spacing between piles for implementing a crop.

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